1. Evaluate the following determinants:

$$a)\ \begin{vmatrix} 1 & i & 1+i \\ 0 & 1 & 2i \\ i-1 & 0 & 0 \end{vmatrix} \qquad b)\ \begin{vmatrix} 1 & -1 & 2 \\ 0 & 3 & 0 \\ -1 & 4 & -2 \end{vmatrix}$$

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2. Given the matrix

$$A = \begin{pmatrix} 1 & -2 & 1 & 3 & 4 \\ 1 & -1 & 0 & 2 & 4 \\ 2 & 1 & 3 & 1 & 2 \\ -1 & 0 & 1 & 1 & 3 \\ 0 & 1 & -1 & 1 & 3 \end{pmatrix}$$

Use Gauss elimination to transform A in an upper triangular matrix
Deduce $|A|$

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3. Let

$$A = \begin{pmatrix} 1 & 2 & 0 \\ -1 & 3 & 0 \\ 0 & 1 & -1 \end{pmatrix}$$

Evaluate $|A|$ and deduce that A is invertible
Find the inverse of A by the cofactors method and then by the Gauss-Jordan method
deduce from a) the determinant of A^{-1} and then calculate directly $|A^{-1}|$

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4. For what values of $a \in IR$ the matrix $A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 3 & a \end{pmatrix} \in M_3(IR)$ is invertible? Find in this case, its inverse .

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5. Show that a square matrix A is invertible if and only if ${}^tA.A$ is invertible
Find the matrix A if $(I+2A)^{-1} = \begin{pmatrix} 2 & 5 & 5 \\ -1 & -1 & 0 \\ 2 & 4 & 3 \end{pmatrix}$

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6. Soit la matrice

$$D_n = \begin{pmatrix} x & a & \cdots & \cdots & a \\ a & x & a & \cdots & a \\ \vdots & a & & & \vdots \\ \vdots & \vdots & & & a \\ a & a & \cdots & a & x \end{pmatrix}$$

En remarquant que la somme des éléments de chaque colonne (resp de chaque ligne) de D_n est $x+(n-1)a$, Calculer le déterminant de D_n

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7. Résoudre l'équation en x

$$\begin{vmatrix} 1 & 1 & \cdots & \cdots & 1 \\ 1 & 1-x & 1 & \cdots & 1 \\ \vdots & & & & \\ 1 & \cdots & \cdots & 1 & n-x \end{vmatrix} = 0$$