Soluț: partiel 2019-2020

a) $L_4 = \{a, a^2, a^3, a^4, b^2a, b^2a^2, ab^2a\}$ $7 \times \frac{1}{2} = 3.5$ pts

b) $$\begin{cases} D_1 = a D_2 + b D_4 \\ D_2 = a D_5 + b D_4 + \varepsilon \\ D_3 = a D_2 + b D_4 \\ D_4 = a D_6 + b D_3 \\ D_5 = a D_5 + b D_4 + \varepsilon \\ D_6 = (a+b) D_6 \end{cases}$$

$6 \times \frac{1}{2} = 3$ pts

$\frac{1}{2} \times 3 = 1.5$ pt

c) $D_6 = \phi$; $D_1 = D_3$ et $D_2 = D_5$

d) $D_5 = a^*(b D_4 + \varepsilon)$ 1pt

$D_4 = b D_3$ 1pt

$D_3 = b^2 D_3 + a D_2$

$D_3 = (b^2)^* a D_2$ 1pt

$D_2 = a^+(b D_4 + \varepsilon) + b D_4 + \varepsilon$

$= (a^+ + \varepsilon) b D_4 + a^+ + \varepsilon$

$= a^* b D_4 + a^*$

$D_2 = a^* b\, b (b^2)^* a D_2 + a^*$

$= a^*(b^2)^+ a D_2 + a^*$

$D_2 = (a^*(b^2)^+ a)^* a^*$ 2pts

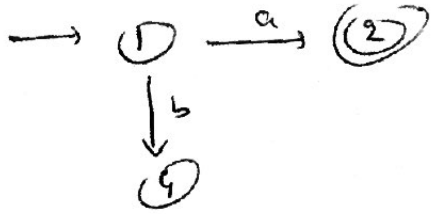
$D_1 = a(a^*(b^2)^+ a)^* a^* + (b^2)^+ a (a^*(b^2)^+ a)^* a^*$ 2pts

$L = (b^2)^* a (a^*(b^2)^+ a)^* a^*$.

2pts

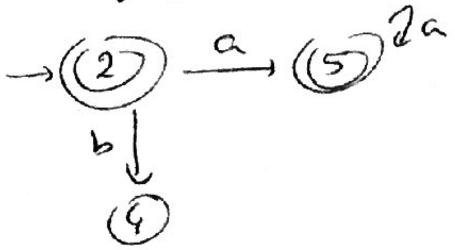
d) 9 pts

e) D_1 est le lagage de l'automate



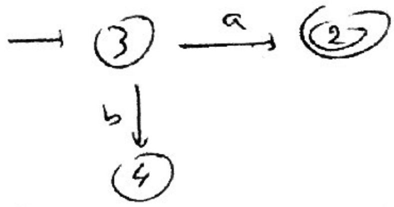
0.5

D_2 est le langage de l'automate



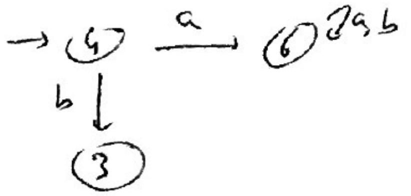
0.5

D_3 est le langage de l'automate



0.5

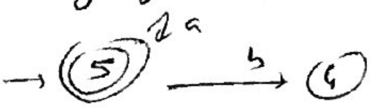
D_4 est le langage de l'automate

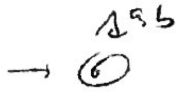


0.5

D_5 est le langage de l'automate

0.5



D_6 est le langage de l'automate  0.5

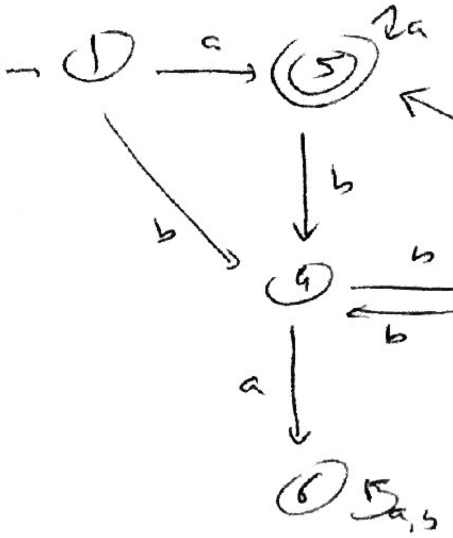
Les mots les plus courts de les langages sont respectivement :

$m_1 = a$; $m_2 = \varepsilon$; $m_3 = a$; $m_4 = \phi$; $m_5 = \varepsilon$; $m_6 = \phi$. $6 \times 0.5 = 3$ pts

on a d'après c) $D_1 = D_3$ et $D_2 = D_5$ donc $q_1 \sim q_3$ et $q_2 \sim q_5$.

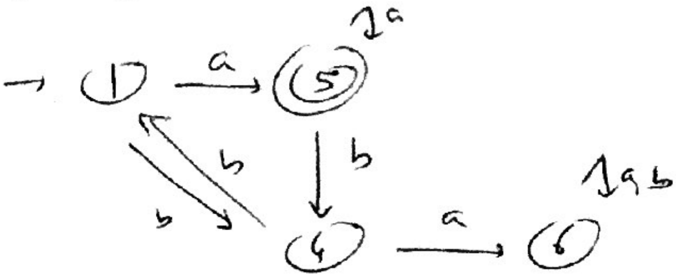
0.5 + 0.5

· $q_2 \sim q_5$ donne :



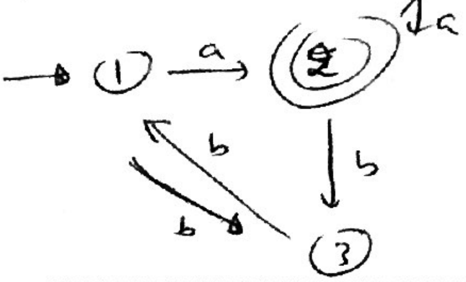
3pts

$q_1 \sim q_3$ donne



3pts

Finalement, l'automate minimal est dans sa forme incomplète ou réduite :

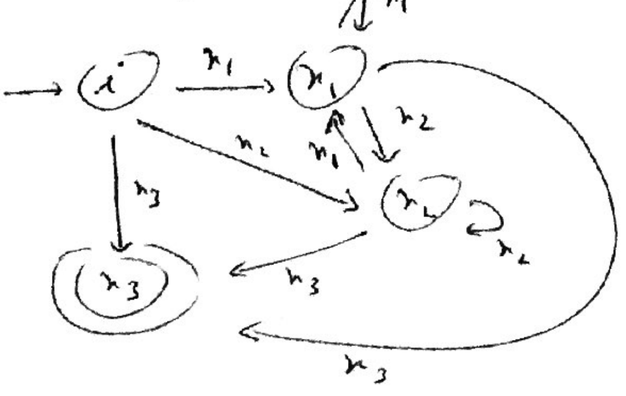


Ex2 : a) $Pr = \{x_1, x_2, x_3\}$ 2pts $\quad (a+b)^*c \equiv (x_1+x_2)^*x_3$ 1pt

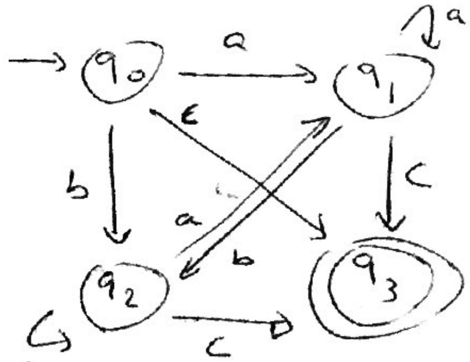
$Dr = \{x_3\}$ 2pts

	Suiv
x_1	x_1, x_2, x_3
x_2	x_1, x_2, x_3
x_3	–

$1.5 + 1.5 = 3$pts



3pts



4pts

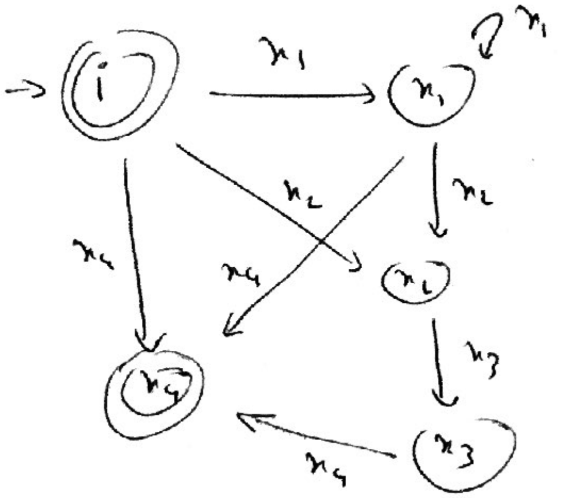
b) $a^*(\varepsilon + bb)a + \varepsilon \equiv x_1^*(\varepsilon + x_2 x_3)x_4 + \varepsilon$ 1pt

$Pr = \{\varepsilon, x_1, x_2, x_4\}$ 2pts

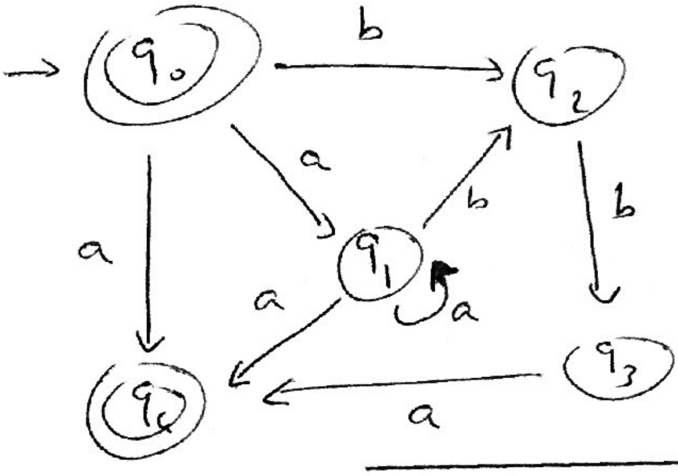
$Dr = \{x_4\}$ 2pts

	Suiv
x_1	x_1, x_2, x_4
x_2	x_3
x_3	x_4
x_4	-

1.5
0.75 = 3pts
0.75



3pts



4pts

Ex3 a) $S \to aSb^2 \to a\underbrace{aSb^2}b^2 \dashrightarrow a^n S (b^2)^n$

$\to a^n (b^2)^n$ et comme $S \to \varepsilon$ alors,

$L = \{a^n (b^2)^n \mid n \geq 0\}$ 5pts

b) $S \to aUc \to a\,aUbc \to a^2\,aUb\,bc$

$\to a\,a^2 U b^2 c \dashrightarrow a\,a^n \varepsilon b^n c$

$L = \{a^{n+1} b^n c \mid n \geq 0\}$. 5pts

Ex4. 5

a) $ba^2 = \varepsilon ba^2 \varepsilon \in a^* b^* a^* b^*$ 1pt

b) $b^* a^* \cap a^* b^* = a^* \cup b^* = \{a^n, b^m \,/\, n, m \geq 0\}$ 1pt

$b^* a^* = \{ b^n a^m \,/\, n, m \geq 0\}$

$= \{ b^n / n \geq 0\} \cup \{a^n / n \geq 0\} \cup \{ b^n a^m \,/\, n, m \geq 1\}$ 1pt

$a^* b^* = \{a^n / n \geq 0\} \cup \{b^n / n \geq 0\} \cup \{a^n b^m \,/\, n, m \geq 1\}.$ 1pt

D'où $b^* a^* \cap a^* b^* = \{a^n, b^m \,/\, n \geq 0, m \geq 0\} = a^* \cup b^*$ 1pt

c) $a^* b^* \cap c^* d^* = \{\varepsilon\} \neq \emptyset$ 1pt

d) abcd est un mot de longueur 4 or les seuls mots de longueur 4 que contient $(a(cd)^*b)^*$ sont :

acdb et abab 2pts

D'où $abcd \notin (a(cd)^*b)^*$.

Ex4: e) $u \in u^*$ donc $(u+v)^* \subset (u^*+v)^*$. 1pt

réciproquement : $u^* \subset (u+v)^*$

$v \subset (u+v)^*$

$u^* + v \subset (u+v)^*$ 1pt

$(u^*+v)^* \subset ((u+v)^*)^* = (u+v)^*$ 1pt

D'où $(u+v)^* = (u^*+v)^*$. 0.5

supplement b) Soit $L = u+v$.

on a $L^* \subset u^* L^*$ car u^* contient $\{\varepsilon\}$.

réciproquement : $u^* \subset (u+v)^* = L^*$

enlevé de l'examen

• donc $u^* L^* \subset L^* L^* = (L^*)^2 = L^*$.

D'où $(u+v)^* = u^*(u+v)^*$.

c) $u+v \subset u+vu^*$ donc $(u+v)^* \subset (u+vu^*)^*$

réciproquement:

$$\begin{cases} u \in (u+v)^* \\ vu^* \subset (u+v)^* \end{cases} \Rightarrow u+vu^* \subset (u+v)^*$$

donc $(u+vu^*)^* \subset \left((u+v)^*\right)^* = (u+v)^*$

D'où $(u+v)^* = (u+vu^*)^*$ → enlevé de l'exam

Ex4

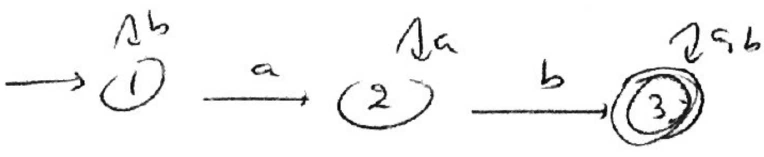
f) $\begin{cases} u \in u^*v^* \\ v \in u^*v^* \end{cases} \Rightarrow (u+v) \in u^*v^*$ (1pt) $\Rightarrow (u+v)^* \subset (u^*v^*)$ (1pt)

aussi $u^*v^* \subset (u+v)^*$ donc $(u^*v^*)^* \subset (u+v)^{**}$

$(u^*v^*)^* \subset (u+v)^*$ (1pt)

D'où $(u+v)^* = (u^*v^*)^*$. (0.5)

Ex5 Soit l'automate A:



10 pts

. Sous la forme complète; son complémentaire

$\bar{A}$:

10 pts

$\bar{L} = \varepsilon + b^*a^+ + b^* = b^*a^*$ 5pts