Université Libanaise | Final | 2011-2012
Faculté des Sciences V | Geométrie Différentielle I (Courbes et Surfaces) | J.SAAB

Durée: 1h

1. (6pts) Let (γ) be the helix giveb byr:

$$\gamma(\theta) = (a\cos\theta, a\sin\theta, b\theta),\ a, b > 0,\ a^2 + b^2 \neq 1 \text{ and } \ \theta \in \mathbb{R}$$

   (a) Verify that γ is not parametrized by arc length
   (b) Calculate the curvature K of (γ)
   (c) Find $s = s(\theta)$ the arc length parameter of γ and refind K, the curvature of γ
   (d) Give the Frenet triefedron of γ at $\gamma(s)$, $(\vec{T}, \vec{N}, \vec{B})$ where $\vec{T}$: is the unit tangent vector to (γ) at $\gamma(s)$; $\vec{N}$: is the normal to (γ) at $\gamma(s)$, $\vec{B} = \vec{T} \wedge \vec{N}$ and calculate the torsion τ of γ
   (e) give a geometric interpretation of your results

---

2. (6pts) Given the smooth function $K : [0, +\infty[\longrightarrow \mathbb{R}$, $K(s) = \dfrac{2}{1+s^2}$

   (a) Find the equation of the plane curve γ who has signed curvature equal to K. (recall that $\cos(2\alpha) = \frac{1-\tan^2\alpha}{1+\tan^2\alpha}, \sin 2\alpha = \frac{2\tan\alpha}{1+\tan^2\alpha}$)
   (b) Can you give an other plane curve $\widetilde{\gamma}(s)$ different from $\gamma(s)$, who has the same signed curvature K

---

3. (8pts) consider the stereographic projection on $S^2 = \{(x, y, z) \in \mathbb{R}^3 \ / \ x^2 + y^2 + z^2 = 1\}$

$$\pi : S^2 - \{P\} \longrightarrow \mathbb{R}^2$$

form S^2 minus the south pole $P(0, 0, -1)$ to $\mathbb{R}^2$ which carries a point $M(x, y, z)$ onto the intersection $m(u, v)$ of the xy plane with the straight line (PM)

   (a) Show that π^{-1} is a chart of S^2 and determine its domain
   (b) Construct an other parametrization of S^2 in order to get S^2, endowed with these parametrizations, as a regular surface

---